Abstract

We analyze a dynamic search and matching model with non-transferable utility and asymmetric information. Randomly paired agents go through an evaluation phase before they discover each other’s types and decide to match or not. Before deciding to enter this phase, agents can send a cheap-talk message about their type to their partner. We provide conditions for this communication to be informative and examine how it impacts search behaviors and the matching that arises in a stationary equilibrium. Early access to truthful information enables agents to avoid spending time in unfruitful evaluation phases and can modify the final matching as it affects how picky agents are. A full characterization of the matching configurations emerging in equilibrium with or without communication is provided. Communication is Pareto improving only when the matching is assortative in the absence of communication and left unchanged by information transmission.

Keywords: Cheap talk; Marriage; Matching; Search.

JEL Classification: C72; C78; D82; D83; J64.
1 Introduction

In matching markets, partners often have limited information about the quality of their match at the time they meet. Yet they also know that, at some point, all information relevant to their match will eventually be revealed. When partners have the same imperfect information at the time of the meeting (as, e.g., in Jovanovic, 1979), they have no other choice than to collect information by being effectively matched for a sufficiently long period. However, most of the time, partners have private information about their match quality that they could communicate right away. Depending on the situation, they would like to transmit this information rapidly or, on the contrary, they may prefer to hide it for as long as possible. For instance, an employer might want to inform quickly a potential employee about the flexibility of working hours offered in his company. Conversely, this employer might prefer to postpone the discussion about how competitive the ambiance in his team can sometimes be. In both cases, a probation period is likely to help the employee discover the true characteristics of the job he might get. Communication during the job interview, through the job opening announcement or at any other preliminary stage of the potential partnership, is an alternative way to learn such characteristics.

This paper builds on the central observation that partners have private information about the match quality at the time they meet and that this information can be elicited by going through a time-consuming evaluation phase. Because agents do not learn the characteristics of their partners right after they met, some room is left for early and voluntary communication between agents. Precisely, we offer individuals the opportunity to communicate via cheap talk, that is, at no cost and without having to prove their statements with hard evidence. A crucial feature of such contexts is that, if such communication is informative, then everything is as if agents could take informed decisions about matching at the time of their first meeting. This paper analyzes the conditions under which information revelation is truthful and its impact on search behavior, equilibrium matching (i.e., who matches with whom) and agents’ welfare.

Formally, we analyze a dynamic search and matching model with non-transferable utility and asymmetric information. Agents can be of two types, high or low, with the same ordinal preference: everyone prefers to be matched with a high-type agent. Before they

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1Incomplete but symmetric information about the match-specific dimension has been studied extensively, both theoretically (see, e.g., Pries, 2004, Pries and Rogerson, 2005), and empirically (see, e.g., Nagypal, 2007).

2Other examples with preliminary communication in matching markets include men and women filling their profiles on dating websites and going through several “dates” before they consider themselves officially together, or researchers organizing informal meetings before starting to work together.
can enjoy any gain from matching, agents have to go through the following process: first, they are randomly paired; second, they communicate bilaterally about their types; third, they decide whether or not to start an evaluation phase during which they cannot search for another partner; last, at the end of the evaluation phase, types are revealed and the informed decision of whether or not to effectively match is taken. We fully characterize the equilibria of this game according to the matching that arises in the steady state and to whether agents truthfully communicate.

When no information is transmitted during the communication phase, decisions to be matched with a known type are taken at the end of the evaluation phase. If communication is informative, everything is as if paired agents immediately learnt each other’s type and therefore took final matching decisions at the time of meeting. In both cases, three possible outcomes can emerge in the steady-state: assortative matching when agents are matched assortatively by types; upward matching when high types accept to match with any other type and low types reject each other; and random matching which corresponds to the initial random pairs being finally matched. Informative communication by itself cannot compensate entirely for search frictions and, therefore, the frictionless complete information outcome (assortative matching) does not obtain systematically under informative communication. Said differently, the same outcomes may arise in both cases but under different conditions.

Truthful information revelation is not trivial in our model as agents communicate about vertical characteristics while all competing for the “best” partners. We show how agents’ incentives to reveal the truth crucially depend on the equilibrium matching configuration that follows. In cases where high types refuse to marry low types, the latter may want to claim being of high type in order to be spotted by high-type agents. However, such a lie reveals unprofitable if it leads a low-type agent to start evaluation phases at the end of which he will be rejected anyway. Indeed, in our model, information transmission can only be used by an agent to convince his partner to start an evaluation phase but nothing prevents the partners from rejecting each other once the truth is known. Our analysis precisely unveils that low-type agents have no interest in lying if high-type agents are patient enough or if high-type agents’ chances to be matched quickly with another high-type are big enough. In particular, this shows that truthful communication does not always obtain in equilibrium.

The model also sheds light on the consequences on the functioning of matching markets of having more information about a potential partner at an early stage of a relationship. The question is of interest as the decline in communication costs has made information transmission much easier in modern matching markets. \(^3\) Everything else being equal, the

\(^3\)A number of papers have discussed the role of information technology in the matching between workers
opportunity cost of continuing to search is lower when communication is informative because agents no longer lose time in unfruitful evaluation phases. Communication therefore provides everyone with stronger incentives to reject lower-types agents. This effect is however less pronounced for the low-type agents since, at the end of the day, they are still constrained by high-type agents’ acceptance decision. Put differently, if earlier information makes everyone want to be pickier, only the more desired partners can indeed be pickier.

On top of this ambiguous effect of informative communication on the final matching configurations, we identify two channels through which such communication affects agents’ welfare. First, as mentioned earlier, agents no longer lose time in evaluation phases with agents they finally reject, which is beneficial for everyone. Second, and this is more subtle, the continuation payoff of being single may not be the same when communication is absent or informative. In particular, we show that, given a matching outcome, informative communication induces search externalities at the time of meeting. By taking earlier informed decisions of whether or not to reject a type, agents affect the steady-state pool of singles in a way that can deteriorate or improve it compared to the case of no information. In the end, this search externality can make it harder or easier to find a good partner. Overall, resulting from the conjunction of three effects, the comparison of welfare under no-communication and informative communication proves quite complex. Yet, we identify necessary and sufficient conditions under which each agents’ type is strictly better off. Mainly, we show that communication is Pareto improving only when the matching is initially assortative and left unchanged by informative communication. Put differently, communication is Pareto improving only when the initial matching is the efficient one.

Related literature. Our paper is related to the literature on search and matching models with heterogeneous agents. The literature separates into two strands depending on whether utility is assumed to be transferable (Sattinger, 1995, Shimer and Smith, 2000, Atakan, 2006) or non-transferable (McNamara and Collins, 1990, Eeckhout, 1999, Burdett and Coles, 1997, Bloch and Ryder, 2000, Chade, 2001 and Smith, 2006). A common assumption in such models is that all relevant information for the matching is revealed right after the two

and firms (see, e.g. Kuhn and Skuterud, 2004, Bagues and Labini, 2009, Nakamura et al., 2009, Stevenson, 2009, Kuhn and Mansour, 2014). For a general discussion of the impact of the Internet on the functioning of labor markets, see Autor (2001) and Kuhn (2003). In the marriage market, Bellou (2013) finds evidence that Internet diffusion contributed significantly to the recent rise in marriage rates among young US couples. Relying on data from a dating website, Hitsch et al. (2010) observes that the matching which arises online is similar to the one predicted by the deferred acceptance algorithm.
agents have met. Put differently, there is no evaluation period and, therefore, no room for communication once agents have made contact. As mentioned earlier, we assume that discovering a partner’s type is time consuming.

The focus of this literature has been on finding conditions under which the matching that arises in the steady state is “similar” to the matching that would arise in the absence of any search frictions. For instance, under NTU and identical ordinal preferences, it is well known that a perfectly assortative matching arises under complete information. Smith (2006) shows that a similar outcome obtains when there are search frictions if the surplus function is log-supermodular in the partners’ types. Our approach is different. We aim at comparing the matching when no information is transmitted to the matching under informative cheap-talk communication. Since our benchmark situation is no longer the frictionless case, we also characterize situations where a matching different from the assortative one arises in equilibrium.

An alternative approach to information transmission in matching markets is to assume that information transmission takes place before the matching occurs. In contrast, in our model information transmission occurs within pairs previously formed at random. Hoppe et al. (2009) and Hopkins (2012) show that there exists a separating equilibrium with assortative matching if agents can invest in costly and publicly observable signals (in the sense of Spence, 1973) before being matched. Comparative statics results then relate prematch signals to the compositions and sizes of the populations. Another type of information transmission is considered in Bilancini and Boncinelli (2013) where agents can disclose costly and certified information about their private skills before being matched. Jacquet and Tan (2007) describe a search and matching model in which agents choose a market place to search in before the matching takes place. With a continuum of market places, perfect segmentation (i.e., the frictionless outcome) obtains: low-type agents do not mingle with high-type agents because competition is too harsh in meeting places packed with high-type agents. We see our paper as complementary. With a small number of market places, perfect separation through market place’s choice cannot obtain and there is room for private bilateral communication.

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4 Chade (2006) is a notable exception where agents only get a partially informative signal about the type of their partner right after they meet. But in this model the signal is exogenous and agents cannot reject their partner after an evaluation phase.

5 Cheap talk was introduced in economic theory by Crawford and Sobel (1982) and Green and Stokey (2007). For a survey, see Sobel (2010).

6 These works are connected to the literature on prematch investment which started with Cole et al. (1992, 2001). Under complete information, agents invest to directly affect their attractiveness to firms, partners or coworkers on the subsequent matching market (see, e.g., Peters and Siow, 2002, Chiappori et al., 2009 and Bhaskar and Hopkins, 2013, Bidner, 2014).
within a market place. The informativeness of cheap-talk communication has also been investigated in directed search models (Menzio, 2007, Kim and Kircher, 2011). Last, when agents have heterogeneous preferences about potential partners, Lee and Schwarz (2007) look at communication to all the agents of the other side of the market before the interviews in a market design perspective.

The paper is organized as follows. Section 2 describes the model. Then, we characterize the equilibria when there is no communication (Section 3) and when communication is informative (Section 4). Section 5 compares the matching configurations that arise and agents’ welfare under no-communication and informative communication. Section 6 concludes and discusses some extensions to our framework.

2 Model

Matching Environment and Preferences. To fix terminology, we use marriage as a metaphor for the matching problem we analyze. We consider one population of an atomless continuum of agents searching for a marriage partner.\(^7\) The size of the population is normalized to one. It is made up of heterogeneous agents: a proportion \(\lambda_h = \lambda \in (0, 1)\) of agents are of high type \(h\) and a proportion \(\lambda_l = 1 - \lambda\) are of low type \(l\). Type is time-invariant and private information to each agent.

Time is discrete and the horizon is infinite. At the beginning of each period, every agent in the market is either single or in an evaluation phase. An evaluation phase consists of a stochastic number of periods within which two agents stay matched with each other, while both are still unmarried. In every period of the evaluation phase, there is a probability \(\beta \in (0, 1)\) that the evaluation phase ends before the end of the current period; with probability \(1 - \beta\) the evaluation phase goes on to the next period.\(^8\)

The marriage decisions take place at the end of the evaluation phase. When two agents get married, they leave the market and are immediately replaced by singles of the same types. This assumption – known in the literature as a “cloning assumption” – ensures the stationarity of the population at the steady state.\(^9\)

\(^7\)Our conclusions apply equally in a two-sided model as long as the populations are symmetric.

\(^8\)It is qualitatively the same to consider a deterministic length of the evaluation phase, but less tractable.

\(^9\)See, e.g., McNamara and Collins (1990), Bloch and Ryder (2000), Adachi (2003), Chade (2006), Jacquet and Tan (2007) and the discussion in Smith (2011). The alternative assumption in the literature is to suppose that, when two agents get married, they stay in the market but, in every period, they split up with some probability and then go back to the market as singles. See Appendix B for an analysis and discussion of this alternative modeling assumption.
is $u_{ij} > 0$ to type $i$ and $u_{ji} > 0$ to type $j$. All agents have identical ordinal preferences: they get a higher payoff when married with a high type rather than with a low-type agent, i.e., $u_{ih} > u_{il}$, for all $i \in \{l,h\}$. The per-period payoff of an unmarried agent, either in an evaluation phase or single, is assumed to be the same regardless of his type, and is normalized to 0. Agents maximize their discounted expected payoffs at the common rate $\delta \in (0,1)$.

**Communication and Trial Strategies.** In the beginning of each period, single agents are randomly matched into pairs. When meeting, they do not immediately observe each other’s type. Instead, within the period where their match occurred, every pair of agents can strategically communicate through direct cheap talk before deciding whether or not to enter an evaluation phase with each other.

Communication takes the following form: each of the two types of agents sends a costless message from the set $\{h,l\}$ to his partner. Precisely, an agent’s steady-state communication strategy is a mapping from the set of his possible types $\{h,l\}$ into the set of available messages $\{h,l\}$. The strategy is babbling if the same message is sent by the two types of agents. If all the agents of a given type send a different message from all agents of the other type, then the communication strategy is fully revealing. Without loss of generality, we consider fully-revealing strategies where type-$h$ agents send message $h$ and type-$l$ agents send message $l$.

After having sent a cheap-talk message and received one from his partner, each agent takes a decision about whether or not to enter an evaluation phase. For each agent at that step, a (steady-state) trial strategy is a mapping from his type and the cheap-talk message he received at that period to a probability to enter or not. We denote by $\tau_{ij} \in [0,1]$ the trial strategy of a type-$i$ agent who got message $j$ from his partner: $\tau_{ij}$ is the probability that type $i$ accepts to enter the evaluation phase after receiving message $j$ from his partner. Both consents to enter the evaluation phase are required for the matched agents to enter the evaluation phase. In case of a refusal, both agents go back to the market as singles. Two agents who have been matched, have exchanged messages and agreed to give a trial to each other finally enter an evaluation phase which ends within that same period with probability $\beta$. It is only at the end of this evaluation phase that both agents learn each other’s types. No information is released as long as the evaluation phase goes on to a subsequent period.

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10 We only study equilibria in pure communication strategies. Characterization of equilibria in mixed communication strategies is conceptually similar but less tractable.

11 Since there is a continuum of agents of each type, this is equivalent to the situation in which a proportion $\tau_{ij}$ of type-$i$ agents accept to enter the evaluation phase after receiving message $j$ from their partner.
Marriage Strategies. In each period, agents whose evaluation phase ends finally discover their partner’s type and then decide simultaneously whether or not to marry. For each player, a (steady-state) marriage strategy then maps his type and the type of his partner into a probability to accept the marriage. We denote by \( \mu_{ij} \in [0, 1] \) the marriage strategy of a type-\( i \) agent who observes the type \( j \) of his partner: \( \mu_{ij} \) is the probability that type \( i \) agrees to marry type \( j \) at the end of an evaluation phase. When one agent refuses to marry, the marriage cannot occur and both agents go back to the market as singles. When both agents mutually agree to marry, they leave the market and are immediately replaced by singles of the same types. Note that it is a weakly dominated strategy for each type to refuse the marriage with a type-\( h \) agents, i.e., \( \mu_{ih} = 1 \) for all \( i \).\(^{12}\) Therefore, there are only three possible matching outcomes in pure strategies: Positive Assortative Matching (PAM) when agents of similar types get married together (\( \mu_{hl} = 0, \mu_{ll} = 1 \)), Random Matching (RM) when agents end up as if they were randomly married (\( \mu_{hl} = 1, \mu_{ll} = 1 \)), and Upward Matching (UM) when low types end up being married only with high types whereas high types marry every type (\( \mu_{hl} = 1, \mu_{ll} = 0 \)).

The timing of the game in a typical period is summarized in Figure 1.

Steady-State Equilibrium. We are looking for steady-state (i.e., time invariant) perfect\(^ {13}\) Bayesian equilibria in a steady-state environment. In every period, the (steady) state of the game is given by \( \langle n_i, n_{ij} : (i, j) \in \{l, h\}^2 \rangle \), where \( n_i \) is the number of single type-\( i \) agents at the beginning of the period and \( n_{ij} = n_{ji} \) is the number of type-\( i \) agents who are in their evaluation phase with a type-\( j \) agent as the period starts.

Both the decisions to give a trial or to marry are taken simultaneously and require mutual consent by the two partners. It follows that if one of the agents refuses the evaluation phase or the union, the other agent is indifferent between accepting it or rejecting it. To resolve this ambiguity, we focus on equilibria in which, if matching increases both agents’ payoffs, then they both accept the evaluation phase and the union.

An equilibrium in which agents’ communication strategy is fully revealing will be called an \textit{informative communication equilibrium}\.\(^ {14}\) Such an equilibrium induces the same outcome

\(^{12}\)This is the key difference with transferable utility (TU) models. Under TU, we would also have to consider existence of stationary equilibria in which type-\( h \) agents only accept type-\( l \) and reject type-\( h \) agents. See Smith (2011) for a comparison of TU and NTU search and matching models.

\(^{13}\)Compared to Nash equilibrium, perfection requires that an agent chooses an optimal marriage decision when he realizes, at the end of an evaluation phase, that his partner lied about his type. This only occurs off the equilibrium path, so Nash equilibrium does not make any sequential rationality restriction at such information sets.

\(^{14}\)Our definition of “communication equilibrium” is rather narrow since we only consider simultaneous
type $i$ in evaluation phase

married; replaced by same type single

randomly matched with single of type $j$

Cheap-talk communication: messages in $\{h,l\}$
Trial strategies: $\tau_{ij} \in [0,1]$ for type-$i$ after message $j$

Evaluation phase starts

Evaluation phase stops

$1 - \beta$

Evaluation phase goes on to the next period

(type revealed)
Marriage strategies:
$\mu_{ij} \in [0,1]$ for type-$i$ facing type-$j$

Figure 1: Timing of a Typical Period.
and must satisfy the same strategic incentive constraints as under complete information, but agents should also have an incentive to reveal their true type when they meet. An equilibrium in which the communication strategy is babbling is called a no-communication equilibrium. It induces the same outcome and is characterized by the same incentive constraints as an equilibrium of the game without communication possibilities.

Comments on the Model. In our model, agents have to go through an evaluation phase to discover their partner’s type. This phase captures the idea, central in our work, that private information is costly to learn. We have chosen to model this learning cost by an opportunity cost: while one goes through an evaluation phase to eventually find out his partner’s type, no parallel search is allowed. An alternative modelling with an explicit payment to learn a type would work similarly, but provide a less smooth analysis in which participation in the market would depend on tedious comparisons between this payment and the marriage payoffs.

We also assume that agents take the marriage decisions at the end of the evaluation phase and only then enjoy match-specific gains. In fact, what really matters is that it takes the evaluation phase for any pair to create benefits. In particular, marriage can be decided at any point of the evaluation phase. For simplicity, we coupled the marriage decision to the end of the evaluation phase because, at the steady state, the only relevant decisions are those taken at the beginning and at the end of this phase.\textsuperscript{15} Besides, we assume that agents get a zero payoff until the end of the evaluation phase. An example of evaluation phase which is compulsory for production is a phase that also consists of a formation or adaptation period. Until the potential recruit is able to perform the task alone, no surplus is created from the match between this recruit and the firm. If agents received type-dependent payoff during the evaluation phase, types could be directly revealed and no room would be left for communication. If the evaluation phase yielded constant positive payoff, incentives to enter an evaluation phase would lie both in learning a partner’s type and in getting some temporary payoffs. Our focus is on the evaluation phase as a period needed to discover types and accordingly produce, not on its potential pecuniary aspect.

\footnote{face-to-face communication in pure strategies.}
\footnote{\textsuperscript{15}At the steady state, an agent who accepts to start an evaluation phase has no incentive to leave his current partner until he discovers her type.}
3 No-Communication Equilibria

We first characterize no-communication equilibrium outcomes, i.e., the different matching configurations that can occur in equilibrium when no information is transmitted through cheap-talk messages. In that case, a single agent always accepts to enter an evaluation phase: $\tau_{ij} = 1$ for every $i, j \in \{h, l\}$. Indeed, agents have no information on which to condition that choice, and we rule out cases in which players coordinated on the dominated equilibrium where no one ever enters an evaluation phase with anyone.

3.1 Steady State

Since agents whose evaluation phase ends either return to the marriage market as singles or are replaced by single agents of the same type, the steady state $\langle n_i, n_{ij} : (i, j) \in \{l, h\}^2 \rangle$ is the same in every no-communication equilibrium. The number of single type-$i$ agents is equal to the number of type-$i$ agents whose evaluation phase ends:

$$n_i = \beta(n_l + n_{ih} + n_i). \quad (1)$$

The number of type-$i$ agents matched with type-$j$ agents in the evaluation phase is equal to the number of such agents whose evaluation phase continues from a period to the next plus the number of single type-$i$ agents that are newly matched with type-$j$ agents:

$$n_{ij} = (1 - \beta) \left( n_{ij} + n_i \frac{n_j}{n_i + n_j} \right). \quad (2)$$

For every $i \in \{l, h\}$, type-$i$ agents in the market are either singles or in an evaluation phase with some type-$j$ agent, so we have: $n_i + n_l + n_{ih} = \lambda_i$. We use this equation to rearrange (1) and (2) and get:

$$n_i = \beta \lambda_i \text{ and } n_{ij} = (1 - \beta) \lambda_i \lambda_j. \quad (3)$$

In particular the proportion of single type-$i$ agents is identical to the overall proportion of type-$i$ agents ($\frac{n_i}{n_l + n_h} = \lambda_i, \ i = l, h$).

3.2 Equilibria

We now characterize all possible matching configurations which emerge in equilibrium in the absence of communication. In the no-communication case, agents decide to accept or reject
a given type at the end of the evaluation phase. At that point of time, agents compare the immediate gain of marriage to the continuation payoff they get when returning to the single status. In what follows, we denote by $V_i$ the continuation equilibrium payoff of a single type-$i$ agent, and by $V_{ij}$ the continuation equilibrium payoff of a type-$i$ agent who is in the evaluation phase with a type-$j$ agent. We will also use the following notation

$$\zeta(x) := \frac{x}{1 - (1-x)\delta},$$

with $x \in (0,1)$, and observe that $\zeta(x) \in (0,1)$ is increasing in $x$ and $\delta$, and $\zeta(xy) < \zeta(x)\zeta(y)$ for all $x, y \in (0,1)$.

In a no-communication PAM equilibrium ($\mu_{hl} = 0, \mu_{ll} = 1$), every type-$l$ agent accepts to marry any other agent independently of his type, while every type-$h$ agent accepts to marry only type-$h$ ones. First, note that if $\mu_{hl} = 0$ (i.e., type-$h$ always reject type-$l$), then in equilibrium type-$l$ agents should indeed always accept type-$l$ agents, i.e., $\mu_{ll} = 1$, as they would obtain 0 in each period otherwise. Therefore ($\mu_{hl} = 0, \mu_{ll} = 1$) is an equilibrium iff type-$h$ agents are not willing to deviate.\(^\text{16}\) A type-$h$ agent who accepts to marry a type-$l$ agent receives a flow payoff $u_{hl}$; if he refuses to marry a type-$l$ agent, he receives his equilibrium continuation payoff $\delta V_h$, where

$$V_h = \lambda \left( \beta u_{hh} + (1-\beta)\delta V_{hh} \right) + (1-\lambda) \left( \beta \delta V_h + (1-\beta)\delta V_{hl} \right) = \lambda \frac{\beta}{1 - (1-\beta)\delta} u_{hh} + (1-\lambda) \frac{\beta}{1 - (1-\beta)\delta} \delta V_h = \zeta(\beta \lambda) u_{hh}. \quad \text{(5)}$$

Therefore, ($\mu_{hl} = 0, \mu_{ll} = 1$) is an equilibrium iff $u_{hl} \leq \delta V_h$, i.e.,

$$\frac{u_{hl}}{u_{hh}} \leq \delta \zeta(\beta \lambda). \quad \text{(6)}$$

The analysis to get the existence conditions of no-communication RM and UM equilibria follows the same logic and is relegated to Appendix A.1. It is also shown that, generically, there is no equilibrium in which agents use mixed marriage strategies. To get an intuition of why, think about an equilibrium where type-$h$ uses the mixed marriage strategy $\mu_{hl} \in (0,1)$. To do so, type-$h$ should be indifferent between marrying a type-$l$ agent or not. From the analysis of PAM above, we can deduce type-$h$ agent’s indifference condition $\frac{u_{hl}}{u_{hh}} = \delta \zeta(\beta \lambda)$

\(^{16}\)Notice that since we are looking at steady-state equilibria and since there is a continuum of players (so that a unilateral deviation does not modify the state of the game), any unilateral deviation in a single period is profitable if and only if the corresponding deviation for all remaining periods is profitable.
which holds only for non generic sets of parameters. Proposition 1 summarizes the results and is illustrated by Figure 2. As can be seen in the figure, the matching is assortative (PAM or RM) in the sense of Smith (2006) when the payoff is log-supermodular, i.e. when $u_{ll}/u_{lh} > u_{hl}/u_{hh}$.\footnote{Proposition 3 in Smith (2006) establishes that log-supermodularity of output is a sufficient condition for assortative matching.}

**Proposition 1.** *Generically, there is a unique no-communication equilibrium. It is such that $\tau_{ij} = 1$ for $i, j \in \{h, l\}$, and

- the matching is Positive Assortative if $\frac{u_{hl}}{u_{hh}} < \delta \zeta (\beta \lambda)$,
- the matching is Random if $\frac{u_{hl}}{u_{hh}} > \delta \zeta (\beta \lambda)$ and $\frac{u_{ll}}{u_{lh}} > \delta \zeta (\beta \lambda)$,
- the matching is Upward if $\frac{u_{hl}}{u_{hh}} > \delta \zeta (\beta \lambda) > \frac{u_{ll}}{u_{lh}}$.

For each matching configuration, players’ incentives are affected by their continuation value of being single when they reject type-$l$ agents, which increases with the common threshold $\delta \zeta (\beta \lambda)$. This threshold is the same whatever the matching configuration because the proportion of single type-$h$ agents ($\lambda$) is not affected by the matching configuration. The intuitions about why $\delta \zeta (\beta \lambda)$ increases with $\beta$ and $\lambda$ are clear: as $\beta$ increases, being single is relatively more attractive than getting married right away since the expected time to the next potential gain of marriage decreases; an increase in $\lambda$ improves the pool of available potential partners, which makes the strategy of rejecting type-$l$ more attractive.

### 4 Communication and Incentives for Truth-telling

Before characterizing equilibria under informative communication, we study the equilibrium matching configurations when each agent knows his potential partner’s type before deciding whether or not to initiate an evaluation phase. That is, we first characterize trial and marriage equilibrium strategies under complete information, as if the communication strategies were assumed to be fully revealing. Next, we study agents’ incentive to reveal their true type before deciding to enter the evaluation phase.

We first make the following important observation: if two agents $i$ and $j$ take the decision to enter the evaluation phase with a strictly positive probability once the true type of each partner has been revealed, then they will decide to marry with probability one at the end of the evaluation phase: $\tau_{ij} > 0 \Rightarrow \mu_{ij} = 1$; and if they never start an evaluation phase ($\tau_{ij} = 0$),
then they never have the opportunity to get married. Put differently, when information is fully revealed, everything is as if the final decision about marriage were made as soon as the two agents meet.

4.1 Steady State

We first describe the dynamics of the complete information game as a function of agents’ strategies. Note that it is without loss of generality (in terms of equilibrium outcomes) to consider trial strategies such that each type of agent always accepts to start an evaluation phase with a type-$h$ agent, i.e. $\tau_{ih} = 1$ for all $i$ (recall that both consents are required to enter an evaluation phase).

The steady-state number of single type-$i$ agents in any period is equal to the number of type-$i$ agents whose evaluation phase ended in the previous period plus the number of single type-$i$ agents from the previous period who did not enter an evaluation phase because they rejected (or were rejected by) the agent to whom they had been matched (see Figure 3 for an illustration of transitions probabilities at the steady state):
\[ n_h = \beta(n_{hl} + n_{hh}) + n_h \left( \frac{n_l}{n_l + n_h} (1 - (1 - \beta)\tau_{hl}) + \frac{n_h}{n_l + n_h} \right), \tag{7} \]

\[ n_l = \beta(n_{ll} + n_{lh}) + n_l \left( \frac{n_l}{n_l + n_h} (1 - (1 - \beta)\tau_{ll}^2) + \frac{n_h}{n_l + n_h} (1 - (1 - \beta)\tau_{hl}) \right). \tag{8} \]

Figure 3: Steady-state transitions probabilities for type-\( h \) agents.

The number of type-\( i \) agents matched with type-\( j \) agents in the evaluation phase is equal to the number of such agents from the previous period who continue their evaluation phase plus the number of single type-\( i \) agents from the previous period who entered an evaluation phase with a type-\( j \) agent:

\[ n_{ij} = (1 - \beta) \left( n_{ij} + n_i \frac{n_j}{n_l + n_h} \tau_{ij} \tau_{ji} \right). \tag{9} \]

For every \( i \in \{ l, h \} \), a type-\( i \) agent in the market is either single or in an evaluation phase with some type-\( j \) agent, so we have: \( n_i + n_{il} + n_{ih} = \lambda_i \). We use this equation to rearrange (7), (8) and (9) and get:

\[ n_h = \beta(\lambda_h - n_h) + n_h \left( \frac{n_h}{n_l + n_h} \beta + \frac{n_l}{n_l + n_h} (1 - (1 - \beta)\tau_{hl}) \right), \tag{10} \]

\[ n_l = \beta(\lambda_l - n_l) + n_l \left( \frac{n_l}{n_l + n_h} (1 - (1 - \beta)\tau_{ll}^2) + \frac{n_h}{n_l + n_h} (1 - (1 - \beta)\tau_{hl}) \right). \tag{11} \]

The proportion of single type-\( h \) agents now depends on the strategies \( \tau_{hl} \) and \( \tau_{ll} \) and on parameters \( \lambda \) and \( \beta \); it is given by

\[ \hat{\lambda}(\tau_{hl}, \tau_{ll}) := \frac{n_h}{n_l + n_h}, \tag{12} \]
where \((n_h, n_l)\) solves (10)–(11). For the case of complete information random matching, we get in particular that \(\hat{\lambda}(1, 1) = \lambda\). We denote \(\hat{\lambda}(0, 1) = \hat{\lambda}_{PAM}\) and \(\hat{\lambda}(1, 0) = \hat{\lambda}_{UM}\). Figure 4 illustrates the values of the complete information proportions of single type-\(h\) under PAM and UM as a function of the proportion of type-\(h\) agents \((\lambda)\). Some useful properties of the steady-state proportion \(\hat{\lambda}(\tau_{hl}, \tau_{ll})\) of single type-\(h\) agents are summarized by Lemma 1 below.

Figure 4: Complete information proportions of single type-\(h\) agents under positive assortative matching \((\hat{\lambda}_{PAM})\) and under upward matching \((\hat{\lambda}_{UM})\) as a function of the proportion of type-\(h\) agents \((\lambda)\).

**Lemma 1.** The steady-state proportion \(\hat{\lambda}(\tau_{hl}, \tau_{ll})\) of single type-\(h\) agents has the following properties:

1. \(\hat{\lambda}_{PAM} \geq \lambda\) iff \(\lambda \leq 1/2\); \(\hat{\lambda}_{PAM} \geq \beta \lambda\) for all \(\lambda\);
2. \(\beta \lambda \leq \hat{\lambda}_{UM} \leq \hat{\lambda}(1, \tau_{ll}) \leq \lambda\) for all \(\lambda\);
3. \(\hat{\lambda}(\tau_{hl}, \tau_{ll})\) is increasing in \(\lambda\).

**Proof.** See Appendix A.3. \(\Box\)

\(^{18}\)See the proof of Lemma 1 for the exact formulas.
Steady-state proportions of singles depend on trial strategies in the case of complete information while they were independent of the matching configurations in the case of no-communication. In both cases, the cloning assumption – the assumption that married agents are replaced by singles of the same types – mutes the feedback effect of marriage strategies on the steady-state pool of singles.\textsuperscript{19} Instead, it enables to isolate the effect that early information has on that pool: in the absence of communication, no early information is revealed and some pairs of agents enter the evaluation phase while they will reject each other once their types will be discovered; in the complete information case, early information prevent such pairs from entering an evaluation phase.

Under upward matching, since type-$h$ agents accept both types, the number of type-$h$ singles is not affected by the information being discovered before the evaluation phase might start. Indeed, both in the no-communication and the complete information cases, the type-$h$ agents directly enter an evaluation phase when they are single. However, the total number of single type-$l$ agents—and hence, the total number of singles—is higher under complete information because a type-$l$ agent is never in an evaluation phase with another type-$l$. These two effects lead unambiguously to the proportion of single type-$h$ agents being smaller under upward matching with complete information than under upward matching without communication, i.e., $\hat{\lambda}_{UM} \leq \lambda$.

Under positive assortative matching, the proportion of single type-$h$ agents with complete information, $\hat{\lambda}_{PAM}$, is higher than $\lambda$ when $\lambda < 1/2$, but it is smaller than $\lambda$ when $\lambda > 1/2$. The intuition of this property is less straightforward, but could be understood as follows. Under positive assortative matching with complete information, both the number of single type-$h$ and type-$l$ are higher compared to a no-communication situation because the evaluation phases involving type-$l$ and type-$h$ agents no longer occur. When $\lambda$ is small ($\lambda < 1/2$), the rise in number of single type-$h$ agents is more pronounced than on number of single type-$l$ agents because, first, there are fewer type-$h$ agents than type-$l$ agents and, second, single type-$h$ agents search only for type-$h$ agents. The opposite logic applies when $\lambda > 1/2$.

\subsection{Complete Information Equilibria}

We now characterize all possible pure and mixed equilibrium trial strategies under complete information. In what follows, $\tilde{V}_{ij}$ denotes the continuation payoff following a deviation by a

\textsuperscript{19} Appendix C demonstrates that, with the alternative assumption that married couples have a probability of divorce, this feedback effect is present even in the absence of communication. We show that it does not qualitatively affect the results but makes the analysis more complex.
single type-\(i\) agent who is in an evaluation phase with a type-\(j\).

Positive assortative matching is an equilibrium under complete information iff type-\(h\) agents are not willing to deviate by entering an evaluation phase with type-\(l\) agents (and hence accept to marry at the end of the evaluation phase). A type-\(h\) agent who accepts a type-\(l\) agent gets

\[
\tilde{V}_{hl} = \beta u_{hl} + (1 - \beta)\delta \tilde{V}_{hl} = \zeta(\beta) u_{hl},
\]

while, if he rejects a type-\(l\) agent, he receives his continuation payoff \(\delta V_h\) where

\[
V_h = \hat{\lambda}_{PAM} \left( \beta u_{hh} + (1 - \beta)\delta V_{hh} \right) + (1 - \hat{\lambda}_{PAM})\delta V_h
= \frac{\hat{\lambda}_{PAM} \beta}{(1 - (1 - \hat{\lambda}_{PAM})\delta)(1 - (1 - \beta)\delta)} u_{hh} = \zeta(\hat{\lambda}_{PAM})\zeta(\beta) u_{hh}.
\]

Therefore, a positive assortative matching equilibrium exists iff \(\delta V_h \geq \zeta(\beta) u_{hl}\), i.e.,

\[
\frac{u_{hl}}{u_{hh}} \leq \delta \zeta(\hat{\lambda}_{PAM}). \tag{14}
\]

The analysis to get the existence conditions of complete information RM and UM equilibria follows the same logic and is relegated to Appendix A.2. Contrary to the case of no-communication, equilibria where agents used mixed trial strategies exist under complete information. To understand why, think about an equilibrium where type-\(h\) uses the mixed strategy \(\tau_{hl} \in (0,1)\). From the analysis of PAM above, we deduce the indifference condition \(\frac{u_{hl}}{u_{hh}} = \delta \zeta(\hat{\lambda}(\tau_{hl}, \tau_{ll}))\). Given that the proportion of singles is now endogenously given by \(\hat{\lambda}(\tau_{hl}, \tau_{ll})\), it is possible to find type-\(l\) agents’ mixed strategies that make type-\(h\) agents generically indifferent. Proposition 2 summarizes the results.

**Proposition 2.** There exists a pure strategy complete information equilibrium such that

- the matching is Positive Assortative iff \(\frac{u_{hl}}{u_{hh}} \leq \delta \zeta(\hat{\lambda}_{PAM})\),
- the matching is Random iff \(\frac{u_{hl}}{u_{hh}} \geq \delta \zeta(\lambda)\) and \(\frac{u_{ll}}{u_{lh}} \geq \delta \zeta(\lambda)\),
- the matching is Upward iff \(\frac{u_{hl}}{u_{hh}} \geq \delta \zeta(\hat{\lambda}_{UM}) \geq \frac{u_{ll}}{u_{lh}}\).

There exists a mixed strategy complete information equilibrium such that

- \((\tau_{hl} = 1, \tau_{ll} \in (0,1))\) iff \(\frac{u_{hl}}{u_{hh}} \geq \delta \zeta(\hat{\lambda}(1, \tau_{ll})) = \frac{u_{ll}}{u_{lh}}\),
- \((\tau_{hl} \in (0,1), \tau_{ll} = 1)\) iff \(\frac{u_{hl}}{u_{hh}} = \delta \zeta(\hat{\lambda}(\tau_{hl}, 1))\) and \(\delta \zeta(\tau_{hl}\hat{\lambda}(\tau_{hl}, 1)) \leq \frac{u_{ll}}{u_{lh}}\),
• \((\tau_{hl} \in (0,1), \tau_{ll} \in (0,1))\) iff \(\frac{u_{hl}}{u_{hh}} = \delta \zeta(\hat{\lambda}(\tau_{hl}, \tau_{ll}))\) and \(\delta \zeta(\tau_{hl}\hat{\lambda}(\tau_{hl}, \tau_{ll})) = \frac{u_{ll}}{u_{lh}}\),

• \((\tau_{hl} \in (0,1), \tau_{ll} = 0)\) iff \(\frac{u_{hl}}{u_{hh}} = \delta \zeta(\hat{\lambda}(\tau_{hl}, 0))\) and \(\delta \zeta(\tau_{hl}\hat{\lambda}(\tau_{hl}, 0)) \geq \frac{u_{ll}}{u_{lh}}\).

All equilibria are such that \(\tau_{ij} > 0 \Rightarrow \mu_{ij} = 1\), and there is no other equilibrium than the ones described above.

The ratios \(\frac{u_{hl}}{u_{hh}}\) and \(\frac{u_{ll}}{u_{lh}}\) for which a pure equilibrium with complete information exists are given by the threshold \(\delta \zeta(\hat{\lambda}(\tau_{hl}, \tau_{ll}))\) while they were given by \(\delta \zeta(\beta \lambda)\) under incomplete information (see Proposition 1). Hence, the thresholds determining the existence of the different matching conditions under complete information depend on \(\beta\) only through the effect of \(\beta\) on the steady-state proportions of singles. This difference is due to the fact that, in the complete information case, choices to accept or reject an agent are made before the evaluation phase starts. An agent therefore compares returning to a single situation to entering an evaluation phase. In both cases, any potential gain of marriage occurs after an evaluation period.

The thresholds determining existence of the different matching conditions under complete information depend on different steady state proportions of singles. It follows that, under complete information, several equilibrium configurations may co-exist.\(^{20}\) In particular, there are some ratios \(\frac{u_{hl}}{u_{hh}}\) and \(\frac{u_{ll}}{u_{lh}}\) for which PAM and UM equilibria co-exist, or PAM and RM equilibria co-exist. However, since we always have \(\zeta(\hat{\lambda}_{UM}) < \zeta(\lambda)\) (see Lemma 1), UM and RM equilibria never coexist.\(^{21}\)

4.3 Informative Communication Equilibria

We now examine the agents’ incentives for telling the truth about their type depending on the matching that arises in the steady state under complete information. Indeed, for an equilibrium of the communication phase to be fully revealing, it has to be that no agent has an interest in misreporting his type when this report is believed and therefore induces the equilibrium trial and marriage strategies of the complete information case.

Recall that the trial strategy \(\tau_{ij}\) is the probability with which a type-\(i\) agent starts an evaluation phase with an agent who claimed to be of type-\(j\). It follows that, a type-\(i\) agent

\(^{20}\)Despite cloning, multiple equilibria may co-exist, which contrasts with existing results in the literature (see, e.g., Bloch and Ryder (2000) or Eeckhout (1999)). This is due to the fact that more information induces search externalities at the time of meeting (\(\hat{\lambda}\) now depends on agents’ strategies).

\(^{21}\)Overall, three distinct orderings of the thresholds are possible: (i) \(\delta \zeta(\hat{\lambda}_{PAM}) < \delta \zeta(\hat{\lambda}_{UM}) < \delta \zeta(\lambda)\) when \(1/2 < \lambda_{PAM} < \lambda_{UM} < \lambda\); (ii) \(\delta \zeta(\lambda_{UM}) < \delta \zeta(\hat{\lambda}_{PAM}) < \delta \zeta(\lambda)\) when \(\lambda_{UM} < 1/2 < \lambda_{PAM} < \lambda\); (iii) \(\delta \zeta(\hat{\lambda}_{UM}) < \delta \zeta(\lambda) < \delta \zeta(\hat{\lambda}_{PAM})\) when \(\lambda_{UM} < \lambda < \lambda_{PAM} < 1/2\).
initially matched with a type- \( k \) first plays equilibrium trial strategy \( \tau_{ij} \) when the type- \( k \) claims to be of type- \( j \), and then plays the equilibrium marriage strategy \( \mu_{ik} \) at the end of the evaluation phase once the truth is known. When communication is truthful, we still have \( \tau_{ij} > 0 \Rightarrow \mu_{ij} = 1 \) as in the complete information case. However, if an agent deviates from truthful revelation at the cheap-talk stage, a pair of agents \( (i, j) \) who do not agree to enter the evaluation phase \( (\tau_{ij} = 0) \) might have to decide whether or not to marry at the end of this phase. That is, even if \( \tau_{ij} = 0 \), the informative communication equilibrium conditions will depend on the off-equilibrium path decisions \( \mu_{ij} = 0 \) or \( \mu_{ij} = 1 \).

Consider first deviation from truth telling in a PAM equilibrium \( (\tau_{hl} = 0, \tau_{ll} = 1) \) (and hence \( \mu_{hl} = 1 \)). If a type- \( h \) deviates from truthful communication and reveals that his type is \( l \), then he will not start an evaluation phase if he is matched with another type- \( h \) agent (who will reject him), and he may start an evaluation phase if he is matched with a type- \( l \) agent. In the first case he is clearly worse off; in the second case the deviation is payoff-relevant only if he deviates from his trial strategy \( \tau_{hl} = 0 \) to \( \tau_{hl} = 1 \), which is not profitable under the complete information equilibrium conditions of the PAM outcome. Now consider a deviation by a type- \( l \) agent, who claims that his type is \( h \). If \( \mu_{hl} = 1 \), then this deviation is profitable because he will start an evaluation phase with a type- \( h \) agent who will accept to marry him at the end of the evaluation phase. Note that, this type- \( h \) would however have rejected a type- \( l \) agent before starting the evaluation phase. Hence, to have truthful communication with a positive assortative matching outcome, a type- \( h \) agent should prefer to reject a type- \( l \) agent at the end of the evaluation phase. Therefore, there is a fully revealing equilibrium with PAM only if

\[
\frac{u_{hl}}{u_{hh}} \leq \delta \zeta(\hat{\lambda}_{PAM}) \zeta(\beta).
\]

Notice that this condition is strictly stronger than the condition for a positive assortative matching equilibrium to exist under complete information (see Condition (14)) since \( \delta \zeta(\beta) \zeta(\hat{\lambda}_{PAM}) < \delta \zeta(\hat{\lambda}_{PAM}) \). Hence, Condition (15) is a necessary and sufficient condition for an informative communication equilibrium with positive assortative matching. This condition states that going on searching should be attractive enough for high types. It is more easily satisfied if high types are patient enough (high \( \delta \)), if the next evaluation phases are not too long (high \( \beta \)), if there are enough single type- \( h \) available (high \( \hat{\lambda}_{PAM} \)), or if the payoff obtained from marrying a high type is relatively high compared to the payoff obtained from marrying a low type (small \( \frac{u_{hl}}{u_{hh}} \)).

Consider now the RM equilibrium \( (\tau_{hl} = \tau_{ll} = 1) \) (and hence \( \mu_{hl} = \mu_{ll} = 1 \)). Since trials
and marriages do not depend on information, truthful communication is clearly incentive compatible.

Next, consider the UM equilibrium \((\tau_{hl} = 1, \tau_{ll} = 0)\) (and hence \(\mu_{hl} = 1\)). Here, the conditions for full information transmission are implied by the equilibrium conditions under complete information: if a type-\(h\) agent pretends to be a type-\(l\) agent, he gets the same payoff as if he told the truth and chose \(\tau_{hl} = 0\) instead of \(\tau_{hl} = 1\); if a type-\(l\) agent pretends to be a type-\(h\) agent, he gets the same payoff as if he told the truth and chose \(\tau_{ll} = 1\) instead of \(\tau_{ll} = 0\). Therefore, there is a fully revealing UM equilibrium iff there is an UM equilibrium under complete information. Exactly the same reasoning applies to the mixed UM equilibrium outcome \((\tau_{hl} = 1, \tau_{ll} \in (0, 1))\).

Finally, consider a mixed equilibrium in which \(\tau_{hl} \in (0, 1)\). Such an equilibrium cannot be implemented with cheap talk because \(\tau_{hl} \in (0, 1) \Rightarrow \mu_{hl} = 1\), that is, if a type-\(h\) agent is indifferent between starting an evaluation phase with a type-\(l\) agent \((\tau_{hl} \in (0, 1))\) then he will strictly prefer to marry him at the end of the evaluation phase \((\mu_{hl} = 1)\). It follows that a type-\(l\) agent will always pretend to be a type-\(h\) in order to start for sure an evaluation phase with a type-\(h\), who will accept him at the end of the evaluation phase.

The following proposition (illustrated by Figure 5) summarizes the informative communication equilibria of the matching game.

**Proposition 3.** There exists an informative communication equilibrium such that

- the matching is Positive Assortative iff \(\frac{u_{hl}}{u_{hh}} \leq \delta \zeta(\beta) \zeta(\hat{\lambda}_{PAM})\),
- the matching is Random iff \(\frac{u_{hl}}{u_{hh}} \geq \delta \zeta(\lambda)\) and \(\frac{u_{ll}}{u_{hh}} \geq \delta \zeta(\lambda)\),
- the matching is Upward iff \(\frac{u_{hl}}{u_{hh}} \geq \delta \zeta(\hat{\lambda}_{UM}) \geq \frac{u_{ll}}{u_{hh}}\),
- the matching is Mixed Upward \((\tau_{hl} = 1, \tau_{ll} \in (0, 1))\) iff \(\frac{u_{hl}}{u_{hh}} \geq \delta \zeta(\hat{\lambda}(1, \tau_{ll})) = \frac{u_{ll}}{u_{hl}}\).

There is no other informative communication equilibrium than the ones described above.

Interestingly, as can be seen on Figure 5, truthful communication does not always obtain in equilibrium. Incentives to transmit information depend crucially on the steady-state matching equilibrium that follows. When this equilibrium is random, upward or mixed upward, informative communication is always possible as no type is rejected by a type he would like to marry. On the contrary, truthful communication is never possible when high-type agents use mixed trial strategies, as low-type agents then have a way to lie that leads to marriage with high types. Finally, when the matching is positive assortative, the possibility
of truthful communication depends on the parameters of the game. In that case, low-type agents are rejected by high types but have no interest to lie if the parameters of the game ensure that staying single is attractive enough for high types (Condition (15)).

5 Informative Communication vs. No-Communication

In sections 3 and 4, we have fully characterized no-communication equilibria and informative communication equilibria. This final section analyzes the welfare effect of information being transmitted at the time agents meet. Said differently, when voluntary communication is truthful, we want to ask which players benefit from it compared to the case of no-communication. We start by providing a detailed analysis of the effect of informative communication on the matching configurations that emerge in equilibrium. We then use this analysis to study welfare changes and give conditions under which communication is Pareto improving.
5.1 Effect of Informative Communication on Matching Outcomes

Agents’ search strategies are affected by informative communication since it advances the time at which they take the informed decisions of whether or not to accept a given type. Under informative communication, informed decisions are taken before an evaluation period may start so that agents do not spend time in unfruitful evaluation phases. It follows that, everything else being equal, the opportunity cost of rejecting the current partner is lower under informative communication, making rejection of low-type agents more appealing. Said differently, informative communication makes every agent want to be “pickier”. The effect of this change in agents’ incentive on equilibrium outcomes formally relies on the detailed comparisons of the thresholds values obtained in Propositions 1 and 3. These comparisons are provided in Appendix A.4 and the possible changes of matching configuration represented in the left part of Table 1.

<table>
<thead>
<tr>
<th></th>
<th>no com.</th>
<th>informative com.</th>
<th>Type-(h) agents (\hat{U}_h - U_h)</th>
<th>Type-(l) agents (\hat{U}_l - U_l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matching Unchanged</td>
<td>PAM → PAM</td>
<td></td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>UM → UM</td>
<td></td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>RM → RM</td>
<td></td>
<td>=</td>
<td>=</td>
</tr>
<tr>
<td>Matching Changed</td>
<td>RM → PAM</td>
<td>+</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td></td>
<td>UM → PAM</td>
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<td>−</td>
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<tr>
<td></td>
<td>RM → UM</td>
<td>−</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>RM → mixed UM</td>
<td>−</td>
<td>+</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Changes in Matching Configurations and Welfare Comparisons.

Interestingly, not all changes in matching configurations are possible when communication becomes informative. In fact, the opportunity of agents to be pickier under informative communication than under no-communication depends on how much desired they are on the market. As type-\(h\) agents are accepted by every type of agent, it is their own decisions that affect their match in equilibrium. It follows that high-type agents always are effectively pickier under informative communication. This implies, for instance, that if the matching is PAM without communication, it cannot switch to another matching configuration once agents communicate truthfully.

For type-\(l\) agents however, the final matching situation depends on whether or not they are accepted by high types. If type-\(h\) agents become strictly pickier with informative communication, it means that type-\(l\) agents are rejected by type-\(h\) agents. Type-\(l\) agents therefore
have no other choice than accepting each other, that is, being less picky. This observation is illustrated by switches from RM or UM to $\hat{PAM}$ where type-$l$ agents have to accept each other under informative communication even if they do not in the absence of communication. On the contrary, if type-$h$ agents do not become strictly pickier with informative communication, then type-$l$ agents have room to become so and reject other low types. According to this observation, a switch from RM to $\hat{UM}$ is possible: type-$h$ agents accept to marry everybody else but type-$l$ reject each other under information communication. However, switches from UM to $\hat{RM}$ or mixed $\hat{UM}$ are impossible as type-$l$ agents would become strictly less picky under informative communication while type-$h$ agents would continue to accept them.

Overall, the previous discussion makes clear that communication cannot make every agent strictly pickier. It follows that truthful information transmission necessarily has an ambiguous effect on how well-matched agents are: if high-type agents are matched with strictly better partners under informative communication, then low-type agents have strictly worse partners.

### 5.2 Welfare Effects of Informative Communication

Our measure of welfare corresponds to the total expected payoff of all type-$i$ agents in a stationary equilibrium:

$$ U_i := n_i V_i + n_{il} V_{il} + n_{ih} V_{ih}, $$

(16)

In assessing whether communication has a positive or a negative impact on individual welfare, there are three effects to consider:

1. First, as pointed out in section 5.1, informative communication may affect search strategies and, therefore, who marries whom. As explained earlier, changes in matching configurations never strictly benefit both type-$h$ and type-$l$ agents in term of match quality.

2. Second, everything else equal, agents no longer spend time in unfruitful evaluation phases when communication is informative. This positive effect of communication benefits everyone.

3. Last, and for a given matching equilibrium, the continuation payoff of being single may not be the same when communication is absent and when it is informative. This stems from the fact that the distribution of types within the steady-state pool of singles may be different in the two situations. This “search externality” effect plays an ambiguous
role since the proportion of high-type agents in the population of singles can be either lower or higher under informative communication than without communication (see Lemma 1). Put differently, communication can make it either easier or more difficult to find a good partner.

The conjunction of these three effects makes welfare comparison difficult. Yet, we are able to identify situations where the comparison is unambiguous. Table 1 summarizes the comparisons of the total expected welfare for each type of agent and for each possible equilibrium matching configuration when one moves from a no-communication equilibrium to an informative communication equilibrium (see A.5 for the detailed computations). These welfare comparisons finally lead to the following proposition.

**Proposition 4.**

- **High-type agents are strictly better off under informative communication iff communication leads to positive assortative matching;**

- **Low-type agents are strictly better off under informative communication iff communication leads to upward matching or if it leads to positive assortative matching without changing the matching configuration.**

In order to understand how the combination of the three effects plays on welfare, let us investigate situations where the matching is $\hat{\text{UM}}$ with informative communication. These situations are interesting because the third effect unambiguously plays negatively for both agents ($\hat{\lambda}_{\text{UM}} < \lambda$): the quality of the pool of singles is deteriorated by early informative communication. It remains to understand the impact of effects 1 and 2. By Table 1, the matching is either UM or RM without communication when it becomes $\hat{\text{UM}}$ with informative communication. Said differently, high-type agents have the same search strategies (accept everyone), whereas low-type agents are (weakly) pickier with informative communication. It follows that effects 1 and 2 are neutral for high-type agents: since they accept everyone both without communication and with informative communication, they do not find better partners and they do not avoid unfruitful evaluation phases under the latter regime. By the third effect, high-type agents are therefore worse off if communication leads to upward matching. Things are more complicated for low-type agents. On the one hand, they find better partners under informative communication if the no-communication matching is RM and similar partners if it is UM: effect 1 is positive or neutral. On the other hand, they no longer spend time in unfruitful evaluation phases with other low-type agents: effect 2 is
positive. Overall, the combination of effects 1 and 2 dominates effect 3 and low-type agents are better off when communication leads to upward matching.

Finally, we ask whether communication can be Pareto-improving. The answer to this question is an immediate corollary of Proposition 4. Quite remarkably, this happens only when the matching is PAM, the frictionless outcome.

**Proposition 5.** Communication is Pareto improving iff the matching is assortative in the absence of communication and left unchanged by truthful communication.

When the matching is positive assortative with and without informative communication, the first effect is neutral. The second effect benefits both types of agents who are assortatively matched more rapidly under informative communication. The third effect is ambiguous: when \( \lambda \leq \frac{1}{2} \), we have \( \hat{\lambda}_{PAM} \geq \lambda \) and therefore single type-\( h \) agents are better off with informative communication, whereas single type-\( l \) agents are worse off. The opposite holds when \( \lambda > \frac{1}{2} \). This possible negative proportion effect of communication on single agents is however compensated by the efficiency gained from avoiding non-assortative evaluation phases.

In the end, what can we expect from improved communication in matching markets? The answer is mixed. On the one hand, as one should suspect, information helps agents to save on search costs. On the other hand, as Proposition 5 establishes, communication is Pareto-improving only when the initial matching is the efficient one (PAM). When the initial matching is inefficient, communication may worsen the situation by making some agents pickier than desirable. Put differently, communication can make non-desirable outcomes easier to sustain.

6 Conclusion

In this article, we propose a model that incorporates cheap-talk communication into a dynamic search and matching model with asymmetric information. The key feature lies in that agents who are randomly paired have to go through a costly evaluation phase to discover each other’s types, and finally take an informed decision to be definitely matched or not. Because agents do not learn directly the type of their partner, some room is left for early information transmission. We show how agents’ incentives to communicate truthfully depend on the matching equilibrium that follows. Only low-type agents have sometimes an interest in lying, which is wiped out if high-type agents refuse to marry them when the truth is discovered at the end of the evaluation phase.
By considering only two types, the model enables to fully characterize the matching configurations that emerge in equilibrium under no communication and informative communication. One main difference between these two regimes is that the opportunity cost of going on searching for a partner is lower under informative communication. Indeed, being informed earlier, agents do not lose time in unfruitful evaluation periods. It follows that informative communication makes every agent want to be pickier than under no communication. We however show that the opportunity of effectively being pickier depends on how desirable an agent is on the market. Next, and despite having well understood how communication changes the equilibrium matches, we explain why assessing its effect on agents’ welfare is complex. Finally, communication proves Pareto improving only when the matching is assortative in the absence of communication and left unchanged by truthful information transmission.

This paper is a first attempt to study the role of cheap-talk communication when discovering a partner’s type is costly. It could be extended in several directions. We have considered a rather elementary opportunity to transmit information as agents communicate only with their partner and about their own type. When a match is not successful, they might transmit to other market participants the information they just acquired about their partner. There could even be room for communication intermediaries who would gather and pass information collected through private search.

A Appendix: Proofs

In what follows, \( V_i \) denotes the continuation equilibrium payoff of a single type-\( i \) agent, and \( V_{ij} \) the continuation equilibrium payoff of a type-\( i \) agent who is in the evaluation phase with a type-\( j \) agent. The corresponding continuation payoffs following an equilibrium deviation by a type-\( i \) agent are respectively denoted by \( \tilde{V}_i \) and \( \tilde{V}_{ij} \).

A.1 Proof of Proposition 1

No-Communication RM Equilibrium \((\mu_{hl} = 1, \mu_{ll} = 1)\). In such an equilibrium, all types of agents accept to marry any type of agent. A type-\( h \) agent who marries a type-\( l \) agent receives a flow payoff \( u_{hl} \); if he rejects a type-\( l \) agent, he receives his continuation payoff \( \delta \tilde{V}_h \). Using a similar calculation as for the no-communication PAM, we get \( \tilde{V}_h = \zeta (\beta \lambda) u_{hh} \).

A type-\( l \) who marries a type-\( l \) receives a flow payoff \( u_{ll} \); if he rejects a type-\( l \), he receives his continuation payoff \( \delta \tilde{V}_l \), where

\[
\tilde{V}_l = \lambda \left( \beta u_{lh} + (1 - \beta) \delta \tilde{V}_{lh} \right) + (1 - \lambda) \left( \beta \delta \tilde{V}_l + (1 - \beta) \delta \tilde{V}_{ll} \right) = \zeta (\beta \lambda) u_{lh}.
\]
Therefore, \((\mu_{hl} = 1, \mu_{ll} = 1)\) is an equilibrium iff \(u_{hl} \geq \delta \tilde{V}_h\) and \(u_{ll} \geq \delta \tilde{V}_l\), i.e.,

\[
\frac{u_{hl}}{u_{hh}} \geq \delta \zeta(\beta \lambda) \quad \text{and} \quad \frac{u_{ll}}{u_{lh}} \geq \delta \zeta(\beta \lambda).
\] (17)

No-Communication UM Equilibrium \((\mu_{hl} = 1, \mu_{ll} = 0)\). In such an equilibrium, every type-\(l\) agent accepts to marry only type-\(h\) agents while every type-\(h\) agent accepts to marry any type of agent. Following the same logic as before, this is an equilibrium iff

\[
\frac{u_{hl}}{u_{hh}} \geq \delta \zeta(\beta \lambda) \quad \text{and} \quad \frac{u_{ll}}{u_{lh}} \leq \delta \zeta(\beta \lambda).
\] (18)

No-Communication Mixed Strategy Equilibria. In a mixed strategy equilibrium where \(\mu_{ij} > 0\) every type-\(i\) agent should be indifferent between marrying a type-\(j\) agent or not. As already mentioned, in any equilibrium we have \(\mu_{hh} = \mu_{lh} = 1\); and from the analysis above, \(\mu_{hl} \in (0,1)\) implies \(\frac{u_{hl}}{u_{hh}} = \delta \zeta(\beta \lambda)\), and \(\mu_{ll} \in (0,1)\) implies \(\frac{u_{ll}}{u_{lh}} = \delta \zeta(\beta \lambda)\). Hence, no-communication mixed equilibria only exist for non generic sets of parameters of the game.

A.2 Proof of Proposition 2

Notice that \((\tau_{hl} = 0, \tau_{ll} = 0)\) is never an equilibrium since type-\(l\) agents always have a strict incentive to start an evaluation phase and get married together when they are rejected by type-\(h\) agents.

Complete Information RM Equilibrium \((\tau_{hl} = 1, \tau_{ll} = 1)\). In this case, the dynamic of the game is the same as in the no-communication equilibrium: the steady-state proportion of type-\(h\) agents in the population of singles is \(\hat{\lambda}(1,1) = \lambda\), and the equilibrium conditions simplify to

\[
\frac{u_{hl}}{u_{hh}} \geq \delta \zeta(\lambda) \quad \text{and} \quad \frac{u_{ll}}{u_{lh}} \geq \delta \zeta(\lambda).
\] (19)

Complete Information UM Equilibrium \((\tau_{hl} = 1, \tau_{ll} = 0)\). Similarly, the conditions under which \((\tau_{hl} = 1, \tau_{ll} = 0)\) is a steady-state equilibrium are given by

\[
\frac{u_{hl}}{u_{hh}} \geq \delta \zeta(\hat{\lambda}_{UM}) \quad \text{and} \quad \frac{u_{ll}}{u_{lh}} \leq \delta \zeta(\hat{\lambda}_{UM}).
\] (20)

Complete Information Mixed Strategy Equilibria. In a mixed strategy equilibrium, if \(\tau_{hl} \in (0,1)\) then a type-\(h\) agent should be indifferent between entering an evaluation phase with a type-\(l\) agent or not. Following the same logic as for the PAM and RM equilibria above, the indifference condition is

\[
\frac{u_{hl}}{u_{hh}} = \delta \zeta(\hat{\lambda}(\tau_{hl}, \tau_{ll})).
\] (21)

If \(\tau_{ll} \in (0,1)\) then a type-\(l\) agent should be indifferent between entering an evaluation phase with a type-\(l\) agent or not. The analysis is slightly more difficult than before, mainly
because $\tau_{hl}$ also appears in $V_l$. Indeed, $\tau_{hl}$ determines the probability that a single type-$l$ agent starts an evaluation phase when he meets a type-$h$ agent. A type-$l$ agent who accepts a type-$l$ agent gets

$$V_{ll} = \beta u_{ll} + (1 - \beta)\delta V_{ll} = \zeta(\beta)u_{ll},$$

while if he rejects a type-$l$ agent he gets $\delta V_{l}$, where

$$V_l = \hat{\lambda} \left( \tau_{hl} \left( \beta u_{lh} + (1 - \beta)\delta V_{lh}\right) + (1 - \tau_{hl})\delta V_{l} \right) + (1 - \hat{\lambda})\delta V_l$$

$$= \frac{\tau_{hl}\hat{\lambda}\beta}{(1 - (1 - \tau_{hl}\hat{\lambda})\delta)(1 - (1 - \beta)\delta)} u_{lh}. \quad (22)$$

Therefore, the indifference condition for $\tau_{ll} \in (0, 1)$ is

$$\frac{u_{ll}}{u_{lh}} = \delta \zeta(\tau_{hl}\hat{\lambda}(\tau_{hl}, \tau_{ll})). \quad (23)$$

### A.3 Proof of Lemma 1

Simplifying Equation (12) yields:

$$\hat{\lambda}(\tau_{hl}, \tau_{ll}) = \beta + (1 - \beta)(\tau_{hl}(1 - 2\lambda) + 2\tau_{hl}^2\lambda) + \sqrt{(\beta + (1 - \beta)\tau_{hl})^2 + 4(1 - \beta)\lambda(1 - \lambda)(\beta + \tau_{hl}^2 - \tau_{hl}(1 - \beta) - 2\tau_{hl}\beta)}$$

$$2(1 - \beta)(1 - \lambda - \tau_{hl}(1 - 2\lambda) - \tau_{hl}^2\lambda).$$

We deduce:

$$\hat{\lambda}_{PAM} = \frac{2\lambda}{2\lambda + \beta(1 - 2\lambda) + \sqrt{\beta^2(1 - 2\lambda)^2 + 4\lambda(1 - \lambda)}},$$

$$\hat{\lambda}_{UM} = \frac{2(1 - \beta)\lambda - 1 + \sqrt{1 - 4(1 - \beta)\lambda(1 - \lambda)}}{2(1 - \beta)\lambda}. \quad (1)$$

1. We have:

$$\hat{\lambda}_{PAM} - \lambda = \lambda(2 - 2\lambda - \beta(1 - 2\lambda) - \sqrt{A})\frac{2\lambda + \beta(1 - 2\lambda) + \sqrt{A}}{2\lambda + \beta(1 - 2\lambda) + \sqrt{A}},$$

where $A = \beta^2(1 - 2\lambda)^2 + 4\lambda(1 - \lambda)$. Since, for all $\lambda \in [0, 1], 2 - 2\lambda - \beta(1 - 2\lambda) \geq 0$ we have

$$\hat{\lambda}_{PAM} \geq \lambda \iff (2 - 2\lambda - \beta(1 - 2\lambda))^2 \geq A.$$

Then, simple calculations show that

$$(2 - 2\lambda - \beta(1 - 2\lambda))^2 - A = 4(1 - \beta)(1 - \lambda)(1 - 2\lambda)$$

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which has the sign of $1 - 2\lambda$. Therefore $\hat{\lambda}_{PAM} \geq \lambda$ iff $\lambda \leq 1/2$. We also have $\hat{\lambda}_{PAM} \geq \beta \lambda$ iff

$$\frac{2}{2\lambda + \beta(1 - 2\lambda) + \sqrt{\beta^2(1 - 2\lambda)^2 + 4\lambda(1 - \lambda)}} \geq \beta$$

$$\Leftrightarrow (2 - \beta(2\lambda + \beta(1 - 2\lambda))^2 \geq \beta^2(\beta^2(1 - 2\lambda)^2 + 4\lambda(1 - \lambda))$$

$$\Leftrightarrow 1 + \beta(1 - 2\lambda)(1 - \beta \lambda) \geq 0,$$

which is always satisfied since $\beta(2\lambda - 1) \leq 1$.

2. To show that $\hat{\lambda}_{UM} \leq \hat{\lambda}(1, \tau_U) \leq \lambda$, it suffices to observe that $\hat{\lambda}(1, 0) = \hat{\lambda}_{UM}$, $\hat{\lambda}(1, 1) = \lambda$ and that

$$\hat{\lambda}(1, \tau_U) = \frac{\sqrt{A} - (1 - 2(1 - \beta)\lambda(1 - \tau_U^2))}{2(1 - \beta)\lambda(1 - \tau_U^2)},$$

where $A = 1 - 4(1 - \beta)\lambda(1 - \lambda)(1 - \tau_U^2)$, is increasing in $\tau_U$. In addition, we have

$$\hat{\lambda}_{UM} - \beta \lambda = \frac{2\lambda(1 - \lambda)(1 - \beta) - 1 + \sqrt{B}}{2(1 - \beta)\lambda},$$

where $B = 1 - 4\lambda(1 - \lambda)(1 - \beta)$. Since, for all $\lambda \in [0, 1]$, $1 - 2\lambda(1 - \lambda)(1 - \beta) \geq 0$, we have

$$\hat{\lambda}_{UM} \geq \beta \lambda \Leftrightarrow B \geq (1 - 2\lambda(1 - \beta)(1 - \beta))^2.$$

Notice that $B - (1 - 2\lambda(1 - \beta)(1 - \beta))^2 = 4\lambda^2(2 - \beta \lambda)(1 - \beta)^2 \geq 0$. Hence, $\hat{\lambda}_{UM} \geq \beta \lambda$.


### A.4 Comparisons of no-communication and informative communication equilibrium conditions

Denote $\frac{\alpha_{uh}}{\alpha_{uh}}$ by $r_h$ and $\frac{\alpha_{uh}}{\alpha_{uh}}$ by $r_l$. To compare the existence conditions of no-communication and informative communication equilibria given by Propositions 1 and 3, we use the two following statements:

(i) $\delta \zeta(\beta \lambda) \leq \delta \zeta(\lambda)$ since $\beta \lambda \leq \lambda$ and $\delta \zeta(\cdot)$ is an increasing function;

(ii) $\delta \zeta(\beta \lambda) \leq \delta \zeta(\hat{\lambda}_{UM}) \leq \delta \zeta(\hat{\lambda}(1, \tau_U))$ as Lemma 1 states $\beta \lambda \leq \hat{\lambda}_{UM} \leq \hat{\lambda}(1, \tau_U)$ for all $\lambda$;

The following list gives the set of parameters for which one can have existence of a no-communication and an informative communication equilibria:

PAM and $\overline{RM}$: from (i), they never coexist since it would require that $r_h = \delta \zeta(\beta \lambda) = \delta \zeta(\lambda)$;

PAM and $\overline{UM}$: from (ii), they never coexist since it would require that $r_h = \delta \zeta(\beta \lambda) = \delta \zeta(\hat{\lambda}_{UM})$;

PAM and mixed $\overline{UM}$: from (ii), they never coexist since it would require that $r_h = r_l = \delta \zeta(\beta \lambda) = \delta \zeta(\hat{\lambda}_{UM})$;

PAM and $\overline{PAM}$: they both exist if $r_h \leq \min\{\delta \zeta(\beta \lambda), \delta \zeta(\beta \hat{\lambda}_{PAM})\}$;

RM and $\overline{RM}$: from (i), they both exist if $r_h, r_l \geq \delta \zeta(\lambda)$;

RM and $\overline{UM}$: from (ii), they both exist if $\delta \zeta(\hat{\lambda}_{UM}) \geq r_l \geq \delta \zeta(\beta \lambda)$ and $r_h \geq \delta \zeta(\hat{\lambda}_{UM})$;
Proof. Let \( \hat{\text{UM}} \) and \( \hat{\text{δζ}} \): from (ii), they both exist if \( r_h \geq \delta \zeta(\lambda(1, \tau_l)) = r_l \); RM and PAM: they both exist if \( \delta \zeta(\beta) \zeta(\hat{\lambda}_{PAM}) \geq r_h \geq \delta \zeta(\beta \lambda) \) and \( r_l \geq \delta \zeta(\beta \lambda) \); UM and RM: from (i), they never coexist since it would require that \( r_l = \delta \zeta(\beta \lambda) = \delta \zeta(\lambda) \); UM and \( \hat{\text{UM}} \): from (ii), they both exist if \( r_h \geq \delta \zeta(\hat{\lambda}_{UM}) \) and \( r_l \leq \delta \zeta(\beta \lambda) \); UM and mixed \( \hat{\text{UM}} \): from (ii), they never coexist since it would require that \( r_h = r_l = \delta \zeta(\beta \lambda) \) and \( r_l \leq \delta \zeta(\beta \lambda) \); UM and PAM: they both exist if \( \delta \zeta(\beta) \zeta(\hat{\lambda}_{PAM}) \geq r_h \geq \delta \zeta(\beta \lambda) \) and \( r_l \leq \delta \zeta(\beta \lambda) \).

A.5 Welfare Comparisons

The next proposition gives the welfare comparisons of the first line of Table 1. Computations for all the other welfare comparisons are relegated to the online appendix.

Proposition 6. The total expected welfare of both low and high type agents is higher in an informative communication PAM equilibrium than in a no-communication PAM equilibrium.

Proof. Let \( \hat{U}_i \) and \( U_i \) denote respectively the type-\( i \) agents total expected welfare in an informative communication and in a no-communication PAM equilibrium. We show that (a) \( \hat{U}_h \geq U_h \) and (b) \( \hat{U}_l \geq U_l \).

(a) \( \hat{U}_h \geq U_h \). We have:

\[
\hat{U}_h - U_h = \hat{n}_{hh} \hat{V}_{hh} + \hat{n}_{hl} \hat{V}_h - n_{hh} V_{hh} - n_{hl} V_{hl}.
\]

Notice that \( \hat{V}_{hh} = V_{hh} = \zeta(\beta) u_{hh} \). Rearranging terms in equation (24), we obtain:

\[
\hat{U}_h - U_h = (\hat{n}_{hh} - n_{hh}) V_{hh} + \hat{n}_{hl} \hat{V}_h - (n_{hh} + n_{hl}) V_h + n_{hl} (V_h - V_{hl}),
\]

\[
= (\hat{n}_{hh} - n_{hh})(V_{hh} - \hat{V}_h) + (\lambda - n_{hh})(\hat{V}_h - V_h) + n_{hl} (V_h - V_{hl}),
\]

where the second equality stems from the fact that \( n_h + n_{hl} + n_{hh} = \lambda \) and \( \hat{n}_{hh} + \hat{n}_{hl} = \lambda \).

Notice that \( V_{hh} - \hat{V}_h \geq 0 \), a type \( h \) is better off being engaged with a type-\( h \) agent than being single, and \( V_h - V_{hl} \geq 0 \), a type \( h \) is better off being single than being engaged with a type-\( l \) agent that he will reject in the end. To show that \( \hat{U}_h - U_h \geq 0 \) we proceed in two steps: we show that (i) \( \hat{n}_{hh} - n_{hh} \geq 0 \) and (ii) \( (\lambda - n_{hh})(\hat{V}_h - V_h) + n_{hl} (V_h - V_{hl}) \geq 0 \).

(i) Recall that

\[
n_{hh} = (1 - \beta) \lambda^2 \text{ and } \hat{n}_{hh} = (1 - \beta) \lambda - \frac{\beta}{2(1 + \beta)} \left( \sqrt{\beta^2 + 4 \lambda (1 - \lambda)(1 - \beta)} - \beta \right),
\]

so that

\[
\hat{n}_{hh} - n_{hh} = \lambda(1 - \lambda)(1 - \beta) - \frac{\beta}{2(1 + \beta)} \left( \sqrt{\beta^2 + 4 \lambda (1 - \lambda)(1 - \beta)} - \beta \right).
\]
Therefore
\[ \hat{n}_{hh} - n_{hh} \geq 0 \iff \left( \frac{2(1+\beta)}{\beta} \lambda (1-\lambda)(1-\beta) + \beta \right)^2 - \beta^2 - 4\lambda (1-\lambda)(1-\beta) \geq 0, \]
which concludes step (i).

(ii) Recall that \( \lambda - n_{hh} = \lambda - (1-\beta)\lambda^2 \) and \( n_{hl} = (1-\beta)\lambda(1-\lambda) \). Tedious but straightforward calculations show that:
\[
(\lambda - n_{hh})(V_h - V_h) + n_{hl}(V_h - V_{hl}) = \lambda \left( (1 - (1 - \beta)\lambda)(\zeta(\beta)(\lambda_{PAM} - \zeta(\lambda)) + (1 - \beta)(1 - \lambda)\zeta(\lambda)(1 - \delta \zeta(\beta))) \right) u_{hh},
\]
where \( A = \beta^2 + 4\lambda(1-\lambda)(1-\beta^2) \). Notice first that \( \sqrt{A} - \beta(2\lambda - 1) \geq 0 \), so that the denominator is positive. This also shows, in particular, that the numerator is (linearly) increasing in \( \delta \). It is equal to \( 2(1 - \lambda) - \beta(\sqrt{A} - \beta(2\lambda - 1)) \) when \( \delta = 0 \). To conclude, notice that:
\[
2(1 - \lambda) - \beta(\sqrt{A} - \beta(2\lambda - 1)) \geq 0 \iff (2(1 - \lambda) + \beta^2(2\lambda - 1))^2 - \beta^2 A \geq 0 \iff 4(1 - \beta^2)(1-\lambda)^2 \geq 0
\]
which always holds.

(b) \( \hat{U}_l \geq U_l \). The proof is similar to part (a). From \( V_{il} = \hat{V}_{il} = \zeta(\beta)u_{il} \) and \( 1 - \lambda = n_i + n_{ih} + n_{il} = \hat{n}_i + \hat{n}_{il} \), we get:
\[
\hat{U}_l - U_l = (\hat{n}_{il} - n_{il})(V_{il} - \hat{V}_i) + (1 - \lambda - n_{il})(\hat{V}_i - V_i) + n_{ih}(V_i - V_{ih}).
\]
In a PAM equilibrium, a type \( l \) is never married with type-\( h \) agents, he is better off being engaged with a type-\( l \) agent than being single, i.e. \( V_{il} - \hat{V}_i \geq 0 \), and he is better off being single than being engaged with a type-\( l \) agent, i.e. \( V_i - V_{ih} \geq 0 \). In the following, we show that (i) \( \hat{n}_{il} - n_{il} \geq 0 \) and (ii) \( (1 - \lambda - n_{il})(\hat{V}_i - V_i) + n_{ih}(V_i - V_{ih}) \geq 0 \).

(i) We have
\[
\hat{n}_{il} - n_{il} = \frac{2(1 - \lambda)\lambda + \beta^2(1 - 2(1 - \lambda)\lambda) - \beta\sqrt{A}}{2(1 + \beta)};
\]
so \( \hat{n}_{il} - n_{il} \geq 0 \iff 4(1 - \beta^2)^2(1 - \lambda)^2 \lambda^2 \geq 0 \), which is always satisfied.
(ii) Tedious but straightforward calculations show that:

\[
(1 - \lambda - n_{ll})(\hat{V}_l - V_l) + n_{lh}(V_l - V_{lh})
= (1 - \lambda) \left( (1 - (1 - \beta)(1 - \lambda))(\zeta(\beta)\zeta(1 - \lambda) - \lambda) \right)
+ (1 - \beta)\lambda \zeta(1 - \delta(\beta)) u_{ll}
= \lambda \zeta(\beta)\zeta(1 - \lambda) \left( \frac{(1 - \delta) \sqrt{A} - \beta(1 - 2(1 - \beta)\delta(1 - \lambda)^2)}{\sqrt{A} - \beta(2\lambda - 1) + 2\lambda(1 - \delta)} \right) u_{ll}.
\]

Notice that \( \beta(1 - 2(1 - \beta)\delta(1 - \lambda)^2) \leq \beta \). Hence, since \( \sqrt{A} \geq \beta \), the numerator is positive. Then, notice that \( \sqrt{A} - \beta(2\lambda - 1) + 2\lambda(1 - \delta) \geq \sqrt{A} - \beta \geq 0 \) which shows that the denominator is positive and concludes the proof.

\[\square\]

B Appendix: Probability of Divorce

In this section we show that our analysis would be similar, but more tedious, when replacing the cloning assumption by the assumption of a probability of divorce. With the cloning assumption, trial strategies have a feedback effect on the steady-state distribution of singles under informative communication but not under no-communication. With a probability of divorce, marriage strategies have such a feedback effect even in the absence of communication. We show that this effect is qualitatively the same as the one of trial strategies under informative communication but is still weaker.

Assume that, once two agents get married, there is an exogenous probability \( \gamma \) that they divorce and go back to the market as singles. In every period, the (steady) state of the game is given by \( \langle n_i, n_{ij}, m_{ij} : (i, j) \in \{l, h\}^2 \rangle \), where \( n_i \) is the number of single type-\( i \) agents at the beginning of the period, \( n_{ij} = n_{ji} \) is the number of type-\( i \) agents who are in their evaluation phase with a type-\( j \) agent as the period starts and \( m_{ij} = m_{ji} \) is the number of type-\( i \) agents who are married with a type-\( j \) agent as the period starts.

Below, we derive the steady-state distribution of agents in no-communication equilibria. The steady-state distribution in informative communication equilibria obtains in the same way. The main difference with the analysis of Sections 3 and 4 is that the steady-state distribution of agents in no-communication equilibria now depends on the marriage strategies. Precisely, given marriage strategies \( \mu_{ij} \), flow equations at the steady state write (see Figure 6 for an illustration of transition probabilities):

\[
n_h = \beta(1 - \mu_{hl}) \frac{n_l}{n_l + n_h} n_h + \beta(1 - \mu_{hl}) n_{hl} + \gamma(m_{hh} + m_{hl}) \tag{28}
\]

\[
n_{hh} = (1 - \beta)n_{hh} + (1 - \beta) \frac{n_h}{n_l + n_h} n_h \tag{29}
\]

\[
m_{hh} = (1 - \gamma)m_{hh} + \beta n_{hh} + \beta \frac{n_h}{n_l + n_h} n_h \tag{30}
\]

\[
n_l = \beta \frac{(1 - \mu_{hl}) n_l + (1 - \mu_{hl}) n_h}{n_l + n_h} n_l + \beta \left( (1 - \mu_{hl}) n_{hl} + (1 - \mu_{hl}) n_{lh} \right) + \gamma(m_{ll} + m_{hl}) \tag{31}
\]
\[ n_{ll} = (1 - \beta)n_{ll} + (1 - \beta)\frac{n_l}{n_l + n_h}n_l \]  
(32)

\[ m_{ll} = (1 - \gamma)m_{ll} + \beta\mu^2 n_{ll} + \beta\mu^2 \frac{n_l}{n_l + n_h}n_l \]  
(33)

\[ n_{hl} = (1 - \beta)n_{hl} + (1 - \beta)\frac{n_l}{n_l + n_h}n_h \]  
(34)

\[ m_{hl} = (1 - \gamma)m_{hl} + \beta\mu h m_{hl} + \beta\mu h \frac{n_l}{n_l + n_h}n_h \]  
(35)

Last, the number of type-\(h\) (\(l\), resp.) agents who are single, in an evaluation phase and married must sum to \(\lambda\) (\(1 - \lambda\), resp.):

\[ n_h + n_{hh} + n_{hl} + m_{hh} + m_{hl} = \lambda \]  
(36)

\[ n_l + n_{ll} + n_{hl} + m_{ll} + m_{hl} = 1 - \lambda \]  
(37)

Rearranging equations (28) to (37), we obtain \(n_l, n_h\) and, therefore, \(\hat{\lambda}(\mu_{hl}, \mu_{ll})\) which is the proportion of single type-\(h\) agents at the steady state.

Figures 7 and 8 compare the proportion of single type-\(h\) agents under PAM and UM in a no-communication equilibrium and in an informative communication equilibrium. Figures 7 and 8 suggest the following:

- \(\lambda_{PAM} \geq \lambda\) iff \(\lambda \leq 1/2\), and \(\hat{\lambda}_{PAM} \geq \lambda\) iff \(\lambda \leq 1/2\);
- \(\lambda_{UM} \leq \lambda\), and \(\hat{\lambda}_{UM} \leq \lambda\);
- \(\hat{\lambda}_{PAM} \geq \lambda_{PAM}\) iff \(\lambda \leq 1/2\);
- \(\hat{\lambda}_{UM} \leq \lambda_{UM}\).

Comparative statics are similar to those obtained under the “cloning” assumption (see Lemma 1 in the main text). The only difference is that, as already stated, the steady-state distribution of singles in no-communication equilibria now depends on marriage strategies. For instance \(\lambda_{UM}\) is no longer equal to \(\lambda\) in UM no-communication equilibria. It should be emphasized that the underlying mechanism that explains this difference is exactly the same as the one that explained why the proportion of type-\(h\) agents in the population of singles was lower under informative communication and under the “cloning” assumption (see the discussion below Lemma 1). Otherwise, early informative communication still makes the steady-state distribution of singles more “responsive” to marriage strategies: 

\[ |\hat{\lambda}_{PAM} - \lambda| \geq |\lambda_{PAM} - \lambda| \] and 
\[ |\hat{\lambda}_{UM} - \lambda| \geq |\lambda_{UM} - \lambda|. \]

---

\(^{22}\)The exact calculations of \(\hat{\lambda}(\mu_{hl}, \mu_{ll})\) and the proofs of the following statements are available upon request.
Figure 6: Steady-state transitions probabilities for type-$h$ agents in no-communication equilibria.
Figure 7: Proportions of single type-$h$ agents under positive assortative matching as a function of $\lambda$ in a no-communication equilibrium (red curve) and in an informative communication equilibrium (blue curve).

Figure 8: Proportions of single type-$h$ agents under upward matching as a function of $\lambda$ in a no-communication equilibrium (red curve) and in an informative communication equilibrium (blue curve).
References


